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1998 J. Phys.: Condens. Matter 10 5807

(http://iopscience.iop.org/0953-8984/10/26/009)

J. Phys.: Condens. Matter **10** (1998) 5807–5820. Printed in the UK PII: S0953-8984(98)88193-9

The reflectivity of a film with a grating in contact with a semi-infinite bimetallic superlattice

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Received 6 October 1997, in final form 17 March 1998

Abstract. We study the influence of a semi-infinite bimetallic superlattice on the 'excitation' of surface polaritons appearing in a film with a grating surface. The transfer-matrix formalism and the Rayleigh–Fano approach are applied to study the optical response for p-polarized light. We analyse the reflectivity and the dispersion relation of the surface plasmon–polaritons, considering Al and Mg for the semi-infinite superlattice and Mg for the film. The 'optical' dispersion relation of surface plasmons is constructed using the minima of the reflectivity. This relation shows a dependence on the corrugation strength such that, as this corrugation grows, the corresponding curve shifts to lower frequencies.

1. Introduction

There has been, for many years, great interest in the study of surface polaritons in different systems, such as semi-infinite homogeneous, laminated, or modulated media [1]. The surfaces of the systems can be either flat or with a surface structure, like gratings or rough surfaces [2–4]. Different theories have been formulated to investigate light scattering from inhomogeneous surfaces. Even though studies of grating surfaces have been carried out using exact theories [5–7], most of the investigations of rough surfaces have been performed using approximate methods.

The diffuse scattering of light and its coupling with surface plasmons at rough surfaces have been studied using the Maxwell equations in terms of the Hertz vector. A coordinate transformation [8] was applied to the homogeneous wave equation yielding a new nonhomogeneous equation, with the nonhomogeneous term depending upon the height of the roughness. The Green function technique was used to solve for the first-order electromagnetic fields scattered by the surface. The Rayleigh–Fano approach and a perturbation method have been employed to study the interactions of light with rough surfaces of conducting [9] and excitonic semiconducting [10] media, in both local and nonlocal theories. The first-order enhancement factors and differential reflectance show resonances due to the coupling of the incident light with the surface plasmons in metals and surface excitons in semiconductors. A theory [11] based on the Ewald–Oseen extinction theorem, and perturbation and Rayleigh–Fano approaches has been applied to the study of surface roughness of metallic media. The results exhibit an enhancement of the scattered electromagnetic fields due to the interaction with surface plasmons. A modal theory has been developed [3, 4] to study grating surfaces and surfaces of modulated semiconductors, to investigate the coupling of electromagnetic waves with surface modes. We consider that this modal theory is suitable for our case.

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In this communication, we apply the modal theory to study the reflectivity $|R_0|^2$ of p-polarized light incident upon a grating surface of a metallic film in contact with a semi-infinite bimetallic superlattice. The Drude model is used to describe the optical response of the materials. We obtain $|R_0|^2$ using the Rayleigh–Fano approach [4] by writing the reflected and refracted fields as superpositions of plane waves that either propagate or decay exponentially as they move away from the sinusoidal grating ruled on the free surface. A small amplitude of the grating ($\xi_0/a \leqslant 0.05$) is considered in order to ensure the validity of the approach in the region $-\xi_0 \leqslant z \leqslant \xi_0$ of the slab, and the transfer-matrix formalism [2] is applied to obtain the electromagnetic fields in the films of the superlattice. Using this procedure and the appropriate boundary conditions permits us to connect the fields of the semi-infinite superlattice with the corresponding ones of the film. It is found that two surface polaritons appear: one arising from the presence of the semi-infinite superlattice and the other existing only due to the grating surface and strongly affected by the corrugation strength.

The simplest macroscopic theory of dielectric response is the Drude theory, so far applied to investigate surfaces of metals. The model may be considered as rather crude, since it ignores charge-density fluctuations, details of surface potentials, quantum interference of electrons, and quantum spilling of electrons into classical forbidden regions. On the other hand, microscopic studies of optical properties of clean and overlayer-covered semi-infinite metals have been carried out using the jellium model [12, 13] to account for the self-consistent electronic density profile. Corrections to the Fresnel formulae are presented in terms of surface response functions such as d-parameters [12] and surface conductivities [14]. The effective-medium theory [15] can be applied to explore electromagnetic wave propagation in conducting superlattices in the long-wavelength regime, but fails above the plasma frequency. Conducting superlattices have artificial periodicity larger than the crystalline period, and no microscopic studies on their electromagnetic response, their normal modes, and their coupling to external probes such as light have been reported to date. The model that we consider in order to study the optical response of rough surfaces of thin films in contact with superlattices is the first attempt to incorporate the effects of artificial materials on the scattering of light by nonhomogeneous surfaces.

The paper is presented as follows. In section 2 we use the transfer-matrix formalism, calculate the dispersion relation for an infinite bimetallic superlattice, and present the Rayleigh–Fano equations corresponding to a metallic film with a grating surface in contact with a semi-infinite superlattice and a system where the superlattice is replaced by a conducting film. Section 3 is dedicated to a discussion, and section 4 presents the conclusions.

2. Formalism

The physical system considered in this report is shown in figure 1. The region $z \leqslant 0$ is occupied by the semi-infinite superlattice made of alternating layers of materials characterized by the frequency-dependent dielectric functions $\epsilon_{\mathrm{A}}(\omega)$, with thickness d_{A}, and $\epsilon_{\mathrm{B}}(\omega)$, with thickness d_{B}, respectively; therefore, the period of the superlattice is $d = d_{\mathrm{A}} + d_{\mathrm{B}}$. The region $0 \leqslant z \leqslant \xi(x) + L$ is occupied by a film with a frequency-dependent dielectric function $\epsilon_F(\omega)$ and thickness $L+\xi(x)$, considering $\xi(x)$ as a sinusoidal profile with period a. Finally, the region $z \geqslant L + \xi(x)$ is filled with vacuum. To solve the problem, we use the transfer-matrix formalism [2] and the Rayleigh–Fano approach [4].

First, we consider the case of the infinite periodic superlattice. For p polarization in layer j (=A or B), the magnetic and electric fields have components of the form

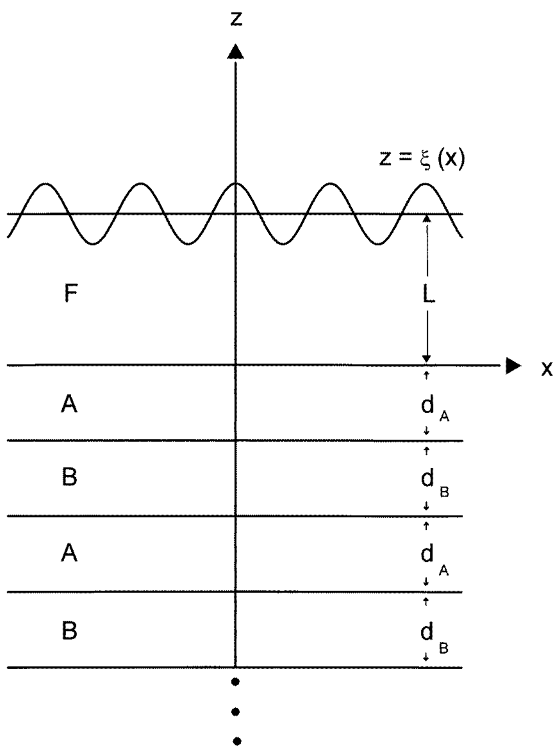

Figure 1. A semi-infinite superlattice made up of a bimetallic unit cell in contact with a corrugated Mg film of thickness L. The layers have dielectric functions $\epsilon_{\mathrm{l}}(\omega)$ and thicknesses d_{l}, $\mathrm{l} = \mathrm{A}, \mathrm{B}$.

$\boldsymbol{H}^{\mathrm{j}}(\boldsymbol{r}, t) = (0, H_2^{\mathrm{j}}(\boldsymbol{r}), 0)\mathrm{e}^{-\mathrm{i}\omega t}$, $\boldsymbol{E}^{\mathrm{j}} = (E_1^{\mathrm{j}}, 0, E_3^{\mathrm{j}})\mathrm{e}^{-\mathrm{i}\omega t}$ which may be written in terms of linear combinations of plane waves, travelling up and down in the film, as

$$H_2^{\mathrm{j}}(x, z|k, \omega) = \sum_p \mathrm{e}^{\mathrm{i}k_p x}\{C_p \mathrm{e}^{-\mathrm{i}\alpha_{p\mathrm{j}} z} + D_p \mathrm{e}^{\mathrm{i}\alpha_{p\mathrm{j}} z}\} \tag{1}$$

and

$$E_1^{\mathrm{j}}(x, z|k, \omega) = \frac{\mathrm{i}c}{\omega\epsilon_{\mathrm{j}}} \frac{\partial H_2^{\mathrm{j}}(x, z|k, \omega)}{\partial z}. \tag{2}$$

Here $k_p = k + (2\pi/a)p$, with $p = 0, \pm 1, \pm 2, \ldots$, and

$$\alpha_{p\mathrm{j}} = \left(\epsilon_{\mathrm{j}}\left(\frac{\omega}{c}\right)^2 - k_p^2\right)^{1/2}.$$

Making use of the transfer-matrix theory we find

$$\begin{pmatrix} H_2^{\mathrm{j}} \\ E_1^{\mathrm{j}} \end{pmatrix}_{z_{\mathrm{j}}^{\mathrm{u}}} (x) = \sum_p \int \mathrm{d}x' \; \mathrm{e}^{\mathrm{i}k_p(x-x')} \mathsf{M}_p^{\mathrm{j}} \begin{pmatrix} H_2^{\mathrm{j}} \\ E_1^{\mathrm{j}} \end{pmatrix}_{z_{\mathrm{j}}^{\mathrm{l}}} (x'). \tag{3}$$

Here the superscripts u and l stand for the upper and lower boundaries of a layer. Using the fact that H_2 and E_1 are continuous across the interfaces of the superlattice, we can write

$$\begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_{z_{\mathrm{A}}^{\mathrm{r}}} (x) = \sum_p \int \mathrm{d}x'' \; \mathrm{e}^{\mathrm{i}k_p(x-x'')} \mathsf{M}_p \begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_{z_{\mathrm{B}}^{\mathrm{l}}} (x''). \tag{4}$$

Taking the Fourier transform of this last equation and using some algebra, we obtain

$$\begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_z (x) = \mathbf{M}_p \begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_{z+d} (x) \tag{5}$$

which is valid for each mode p with $\mathbf{M}_p = \mathbf{M}_p^{\mathrm{A}} \mathbf{M}_p^{\mathrm{B}}$ and

$$\mathbf{M}_{p\mathrm{j}} = \begin{pmatrix} \cos(\alpha_{p\mathrm{j}} d_{\mathrm{j}}) & -\mathrm{i}(\omega\epsilon_{\mathrm{j}}/c\alpha_{p\mathrm{j}})\sin(\alpha_{p\mathrm{j}} d_{\mathrm{j}}) \\ -\mathrm{i}(c\alpha_{p\mathrm{j}}/\omega\epsilon_{\mathrm{j}})\sin(\alpha_{p\mathrm{j}} d_{\mathrm{j}}) & \cos(\alpha_{p\mathrm{j}} d_{\mathrm{j}}) \end{pmatrix}. \tag{6}$$

Now, by means of the Bloch theorem, we write for the infinite superlattice

$$\begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_z (x) = \mathrm{e}^{-\mathrm{i}qd} \begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_{z+d} (x). \tag{7}$$

If we combine equations (5) and (7) we obtain the eigenvalue equation

$$\mathbf{M}_p \begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_{z+d} (x) = \mathrm{e}^{-\mathrm{i}qd} \begin{pmatrix} H_2 \\ E_1 \end{pmatrix}_{z+d} (x). \tag{8}$$

From this equation we obtain the dispersion relation for the collective normal modes given by the condition

$$\det(\mathbf{M}_p - \mathbf{1}\mathrm{e}^{-\mathrm{i}qd}) = 0. \tag{9}$$

Here **1** is the unit matrix.

The magnetic fields in vacuum and in the corrugated film are

$$H_2^v(x, z|k, \omega) = \mathrm{e}^{\mathrm{i}kx - \alpha_{0v}(k,\omega)z} + \sum_p R_p(k, \omega)\mathrm{e}^{\mathrm{i}k_p x + \mathrm{i}\alpha_{pv}(k,\omega)z} \tag{10}$$

$$H_2^F(x, z|k, \omega) = \sum_p \mathrm{e}^{\mathrm{i}k_p x}\{E_p(k, \omega)\mathrm{e}^{-\mathrm{i}\beta_p(k,\omega)z} + F_p(k, \omega)\mathrm{e}^{\mathrm{i}\beta_p(k,\omega)z}\}. \tag{11}$$

In equation (10), we assume that the amplitude of the incident field is equal to one; k is the x-component of the wave vector of the incident light, $k = (\omega/c)\sin\theta$. The z-components of the wave vectors are

$$\alpha_{pv}(k, \omega) = \begin{cases} \left(\left(\dfrac{\omega}{c}\right)^2 - k_p^2\right)^{1/2} & k_p^2 \leqslant \left(\dfrac{\omega}{c}\right)^2 \\ \mathrm{i}\left(k_p^2 - \left(\dfrac{\omega}{c}\right)^2\right)^{1/2} & k_p^2 \geqslant \left(\dfrac{\omega}{c}\right)^2 \end{cases} \tag{12}$$

and

$$\beta_p(k, \omega) = \left(\epsilon_F(\omega)\left(\frac{\omega}{c}\right)^2 - k_p^2\right)^{1/2} \tag{13}$$

with $\mathrm{Re}(\beta_p) \geqslant 0$ and $\mathrm{Im}(\beta_p) \geqslant 0$. We can write the boundary conditions for the corrugated surface in the form

$$H_2^v\big|_{z=L+\xi(x)} = H_2^F\big|_{z=L+\xi(x)} \tag{14}$$

$$\frac{1}{\epsilon_v}\frac{\partial H_2^v}{\partial n}\bigg|_{z=L+\xi(x)} = \frac{1}{\epsilon_F(\omega)}\frac{\partial H_2^F}{\partial n}\bigg|_{z=L+\xi(x)} \tag{15}$$

where $\partial/\partial n$ denotes [1] the derivative along the unit vector normal to the surface $z = \xi(x)$.

Next, we consider the Rayleigh–Fano approach, so that the solutions given by equations (10) and (11) can be continued into the selvedge region $z = \xi(x)$. If we substitute equations (1), (10), and (11) into the boundary conditions and use the periodicity of the

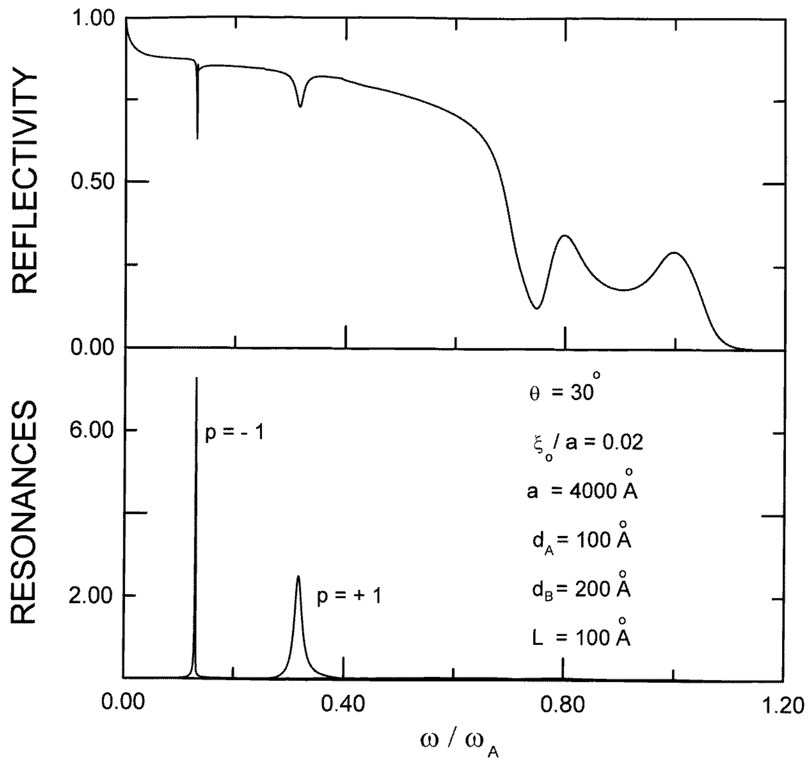

Figure 2. The reflectivity of a Mg corrugated film of thickness $L = 100$ Å in contact with a semi-infinite superlattice made up of Al and Mg layers. The lower panel shows the resonances with $p = \pm 1$.

grating, we obtain a set of equations for the coefficients $R_p(k,\omega)$ and $E_p(k,\omega)$ which can be written in a matrix form [3]:

$$\sum_{p=-\infty}^{\infty}\begin{pmatrix} P_{mp}(k,\omega) & Q_{mp}(k,\omega) \\ T_{mp}(k,\omega) & U_{mp}(k,\omega)\end{pmatrix}\begin{pmatrix} R_p(k,\omega) \\ E_p(k,\omega)\end{pmatrix} = \begin{pmatrix} V_m(k,\omega) \\ W_m(k,\omega)\end{pmatrix} \tag{16}$$

with $m = 0, \pm 1, \pm 2, \ldots$. Here the matrix elements have the forms

$$P_{mp}(k,\omega) = \mathrm{e}^{\mathrm{i}\alpha_{pv}L} X_{m-p}(\alpha_{pv}(k,\omega)) \tag{17}$$

$$Q_{mp}(k,\omega) = -\left[\mathrm{e}^{-\mathrm{i}\beta_p L} X_{m-p}(-\beta_p(k,\omega)) + \mathrm{e}^{\mathrm{i}\beta_p L} X_{m-p}(\beta_p(k,\omega)) S_p(k,\omega)\right] \tag{18}$$

$$T_{mp}(k,\omega) = \mathrm{e}^{\mathrm{i}\alpha_{pv}L}\left[\frac{(\omega/c)^2 - k_p k_m}{\alpha_{pv}} X_{m-p}(\alpha_{pv}(k,\omega))\right] \tag{19}$$

$$U_{mp} = U1_{mp}\left[\mathrm{e}^{-\mathrm{i}\beta_p L} X_{m-p}(-\beta_p(k,\omega)) - \mathrm{e}^{\mathrm{i}\beta_p L} X_{m-p}(\beta_p(k,\omega)) S_p(k,\omega)\right] \tag{20}$$

where

$$U1_{mp}(k,\omega) = \frac{\epsilon_F(\omega)(\omega/c)^2 - k_p k_m}{\epsilon_F(\omega)\beta_p(k,\omega)} \tag{21}$$

$$V_m(k,\omega) = -\mathrm{e}^{-\mathrm{i}\alpha_{0v}L} X_m(-\alpha_{0v}(k,\omega)) \tag{22}$$

$$W_m(k,\omega) = \mathrm{e}^{-\mathrm{i}\alpha_{0v}L}\left(\frac{(\omega/c)^2 - k_0 k_m}{\alpha_{0v}}\right) X_m(-\alpha_{0v}(k,\omega)) \tag{23}$$

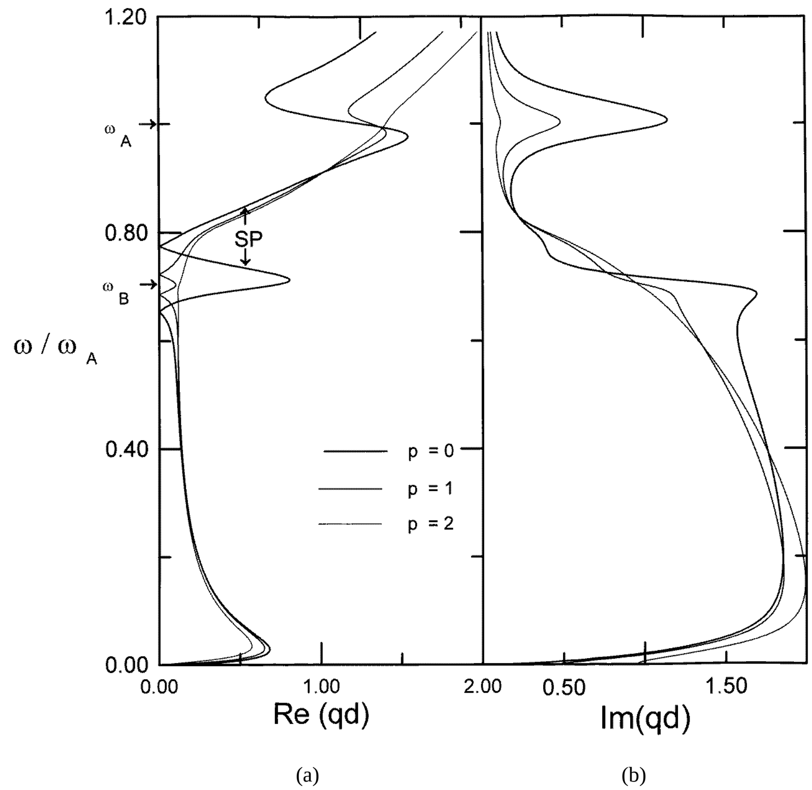

Figure 3. The dispersion relation of the collective normal modes of the infinite superlattice, considering p-polarized waves propagating in the system. Three values of p, namely 0, 1, and 2, have been considered to obtain three different curves. Here $\theta = 30^\circ$, $d_A = 100$ Å, $d_B = 200$ Å, $L = 100$ Å, and $a = 4000$ Å. In (a) we show the real part and in (b) the imaginary part of q, the one-dimensional Bloch wave vector.

$$X_n(\alpha) = \frac{1}{a}\int_{-a/2}^{a/2} \mathrm{d}x \ \exp\left(\mathrm{i}\alpha\xi(x) - \mathrm{i}\frac{2\pi}{a}nx\right). \tag{24}$$

In this case

$$S1_p(k,\omega) = 1 - \frac{\alpha_{pA}(k,\omega)\epsilon_F(\omega)}{\beta_p(k,\omega)\epsilon_A(\omega)} + \left(1 + \frac{\alpha_{pA}(k,\omega)\epsilon_F(\omega)}{\beta_p(k,\omega)\epsilon_A(\omega)}\right)\frac{D_p}{C_p} \tag{25}$$

$$S2_p(k,\omega) = 1 + \frac{\alpha_{pA}(k,\omega)\epsilon_F(\omega)}{\beta_p(k,\omega)\epsilon_A(\omega)} + \left(1 - \frac{\alpha_{pA}(\omega)\epsilon_F(\omega)}{\beta_p(k,\omega)\epsilon_A(k,\omega)}\right)\frac{D_p}{C_p} \tag{26}$$

$$S_p(k,\omega) = \frac{S1_p(k,\omega)}{S2_p(k,\omega)}. \tag{27}$$

The quantity D_p/C_p can be obtained [3, 4] from equation (8) at $z = 0$. Equations (16) form an infinite system; in practice solutions can be obtained by imposing on the indices m and p some finite upper limit N; then the sums for m and p in equation (16) are limited to the range $-N, -N+1, \ldots, 0, \ldots, N-1, N$. In this way, the reflectivity $|R_0|^2$ is calculated, and the choice of N depends upon the convergence of $|R_0|^2$—that is, whether it reaches a point where its value suffers no changes even though N takes larger values.

In order to see how much the reflectivity of light from the grating surface is affected

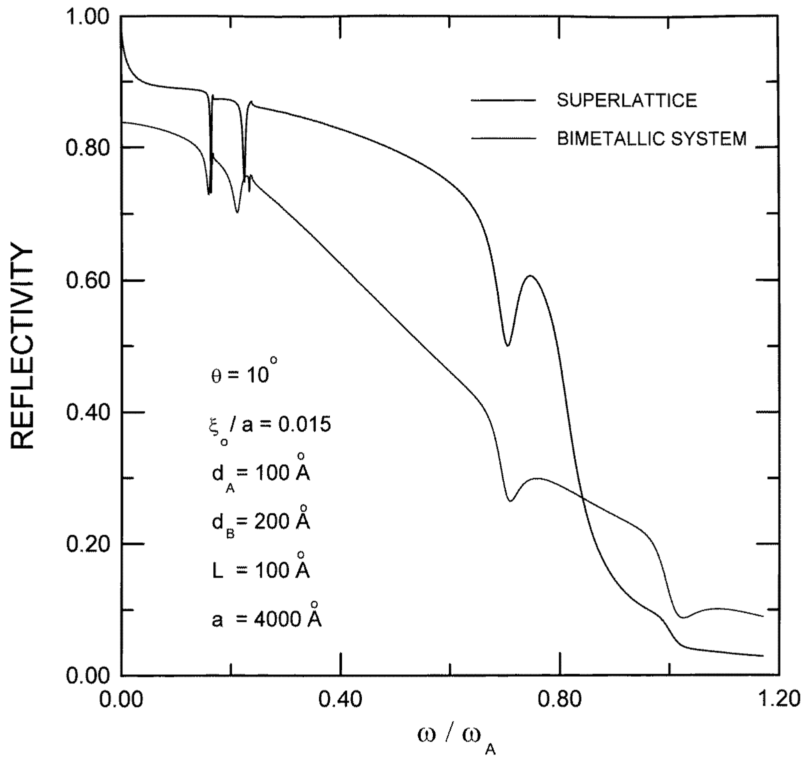

Figure 4. Comparison of the reflectivities of a metallic film with a corrugated shape on the free surface in contact with a flat conducting slab (thin curve) and a semi-infinite superlattice (thick curve). $\theta = 10^\circ$ and $\xi_0/a = 0.015$; the other parameters are given in the figure.

by the semi-infinite superlattice, we compare it with the reflectivity of a system formed by two slabs, one of them with a grating on the free surface. For obtaining a solution for this case, we use again the Rayleigh–Fano approach [4] for the transmitted waves, which can be written as follows:

$$H_2^T(x, z|k, \omega) = \sum_p T_p \mathrm{e}^{\mathrm{i}(k_p x - \alpha_{pv} z)} \tag{28}$$

$$E_1^T(x, z|k, \omega) = \frac{\mathrm{i}c}{\omega \epsilon_v} \frac{\partial H_2^T(x, z|k, \omega)}{\partial z}. \tag{29}$$

Following the previous procedure, we arrive at a system of equations similar to equation (16), but now

$$\frac{D_p}{C_p} = \left(\frac{\gamma - 1}{\gamma + 1}\right) \mathrm{e}^{2\mathrm{i}\alpha_{pa} d_\mathrm{A}}$$

and

$$\gamma = \left(\frac{\epsilon_v}{\epsilon_\mathrm{A}}\right)\left(\frac{\alpha_{p\mathrm{A}}}{\alpha_{pv}}\right).$$

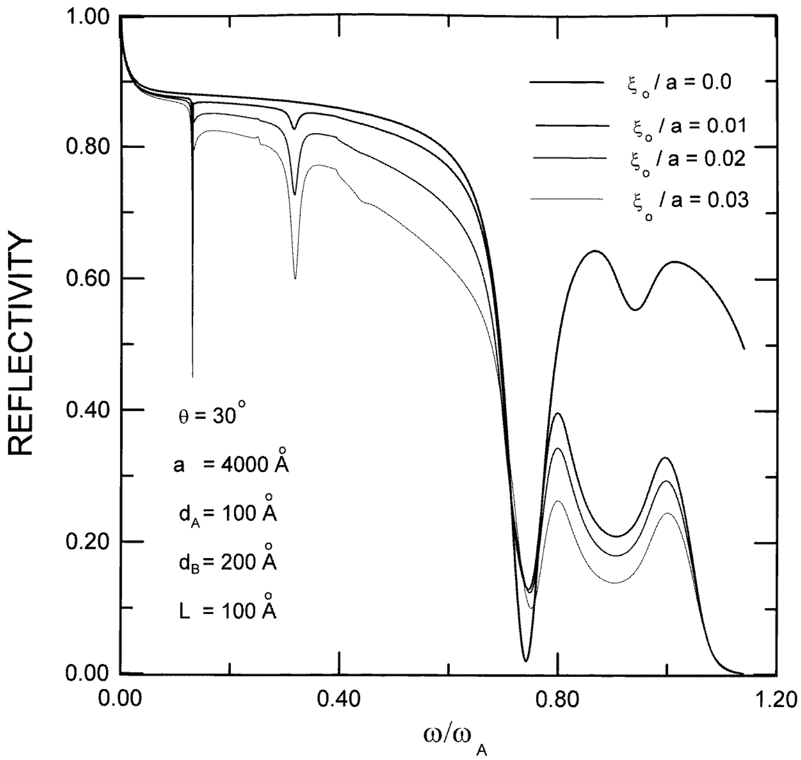

Figure 5. The reflectivity for the system of figure 1 as a function of the corrugated strength ξ_0/a. The other parameters are given in the figure.

3. Results and discussion

We present the results for the reflectivity of p-polarized light incident on a Mg slab with a sinusoidal grating on the free surface in contact with a flat semi-infinite bimetallic superlattice made up of Al and Mg. L is the thickness of the slab, and Al and Mg have thicknesses d_A and d_B, respectively. The dispersion relation for the surface plasmon–polaritons is also calculated as a function of the roughness. We assume that the dielectric function of film l is given by the Drude model:

$$\epsilon_l(\omega) = 1 - \frac{\omega_l^2}{\omega(\omega + i\nu_l)}$$

with $l = A, B$. Here ω_l is the plasma frequency and ν_l is the phenomenological damping term. The parameters relating to Al (film A) and Mg (film B) have the values $\omega_B/\omega_A = 0.7$, $\nu_B/\omega_A = 0.042$, $\nu_B/\omega_A = 0.06$, and $\hbar\omega_A = 15.19$ eV [2], and the periodic grating is described by

$$\xi(x) = \xi_0 \cos\left(\frac{2\pi}{a}x\right)$$

so the $X_n(\alpha)$ are Bessel functions of the second kind. Throughout the calculations, we have considered $a = 4000$ Å as the period of the grating.

In figure 2, we show the reflectivity $|R_0|^2$ in the upper panel and the corresponding resonances $|R_{+1}|^2$ and $|R_{-1}|^2$ in the lower panel. For this figure, $\xi_0/a = 0.02$ and the

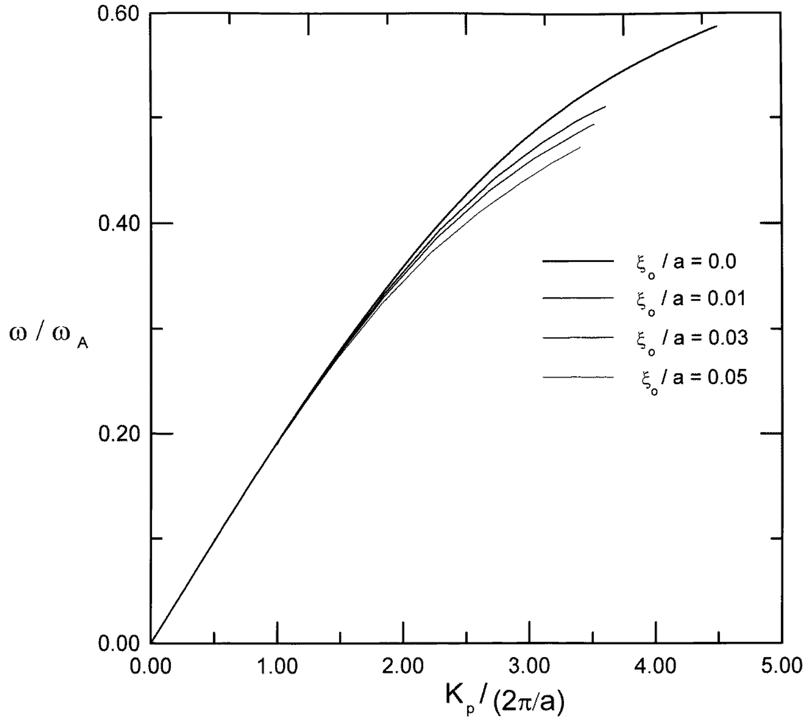

Figure 6. The dispersion relation of the surface plasmon–polariton modes with $p = \pm 1$ obtained from the minima of the reflectivity with the parameters of figure 5 for different values of ξ_0/a.

thicknesses of the films are $d_A = 100$ Å, $d_B = 200$ Å; $L = 100$ Å with an angle of incidence of 30°. The four minima present in the reflectivity are interpreted as follows: the minima at the frequency $\omega = 0.1\omega_A$ and $\omega = 0.3\omega_A$ are due to the surface plasmon–polariton excited by the coupling between the p-polarized light and the surface mode on the metallic film. The minima between the frequencies $0.7\omega_A$ and $1.0\omega_A$ are interpreted in terms of the structure exhibited by the dispersion relation of the collective normal modes for an infinite bimetallic superlattice. These dispersion relations are shown in figure 3 as calculated considering three different values of p (=0, 1, 2); therefore three curves are obtained. The figure displays the real and imaginary parts of the Bloch wave vector q. There, we indicate the surface plasmon–polariton branches (the curve with $p = 0$) which are due to the coupling of the symmetric and antisymmetric modes of the successive sandwich interface modes, in the frequency region $\omega_B \leqslant \omega \leqslant \omega_A$. The structure shown by the curve with $p = 0$ is in correspondence with the behaviour of the reflectivity, as it should be.

In order to corroborate the view that the low-frequency minima displayed by the reflectivity of the system with the superlattice correspond to the surface plasmons excited at the rough surface of the film, we compare with the corresponding result obtained when the superlattice is replaced by a single metallic film. In figure 4, we present the results for the case where the angle of incidence is $\theta = 10°$ and $\xi_0/a = 0.015$. We notice that the reflectivity of the bimetallic system shows two minima at the same frequency as the system with the superlattice, which correspond to the surface plasmons, and two additional minima that resemble the case for the formation of the 'symmetric' and 'antisymmetric' plasmon coupling of the surface modes in a symmetric geometry.

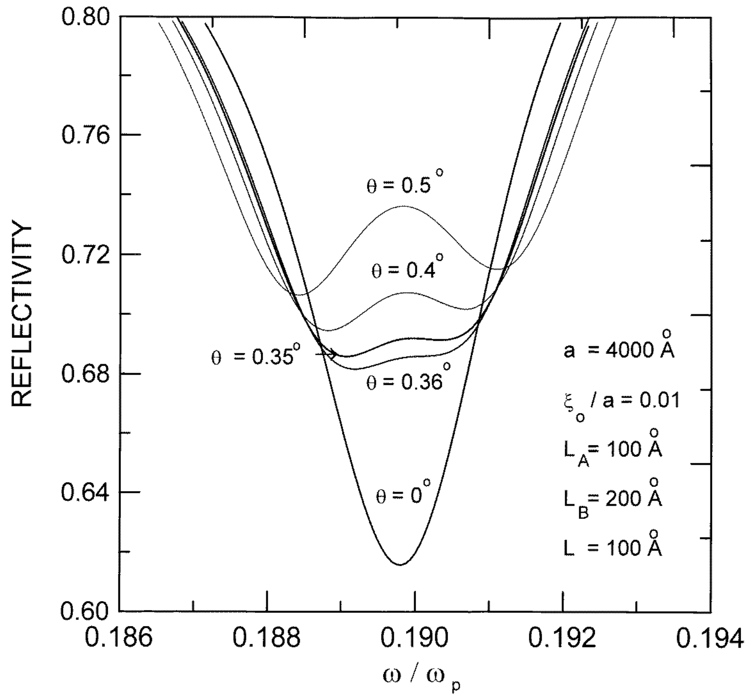

Figure 7. The reflectivity for the system of figure 1 as a function of the angle of incidence θ for $\xi_0/a = 0.01$. The figure shows the behaviour of the minima for near-normal incidence of light.

Figure 5 presents the variations of the reflectivity as a function of the roughness for the parameters of figure 2. We observe how the surface mode minima are very sensitive to the metallic film corrugation strength. As ξ_0/a increases, the two low-frequency minima corresponding to the excitation of the surface polariton become deeper, indicating a strong coupling of the incident light with the surface modes via the grating surface. These minima disappear as the corrugation strength goes to zero. The other minima appearing in the figure show the behaviour of the interface plasmon coupling that forms a mechanism of energy transportation into the bulk of the superlattice. In addition to variations of ξ_0/a, we have considered the limiting case of $L \to \infty$ to reproduce the semi-infinite medium. Numerical calculations demonstrated that $L = 20\,000$ Å is an appropriate limit.

To complement the discussion on the surface plasmons, in figure 6 we depict the 'optical' dispersion relation of the surface polaritons for the parameters used in figure 2. The curves are obtained from the minima of the reflectivity, corresponding to the resonances with $p = \pm 1$, for different values of the corrugation strength. We see that the dispersion relations of these modes shift to lower frequencies as ξ_0/a becomes large and approaches the light line as the corrugation vanishes. Studies of surface plasmons have shown minigaps in the dispersion relation at the boundaries of the Brillouin zones, with their size being a few meV and depending upon the zone boundary. Weber and Mills [7] argue that the appearance of such minigaps is a consequence of the grating symmetry, such that when a symmetric profile is considered, minigaps vanish with decreasing angle of incidence. In contrast, in the asymmetric profile, this minigap is preserved even for normal incidence of light. Both theoretical results are in agreement with experimental evidence from Chen *et al*

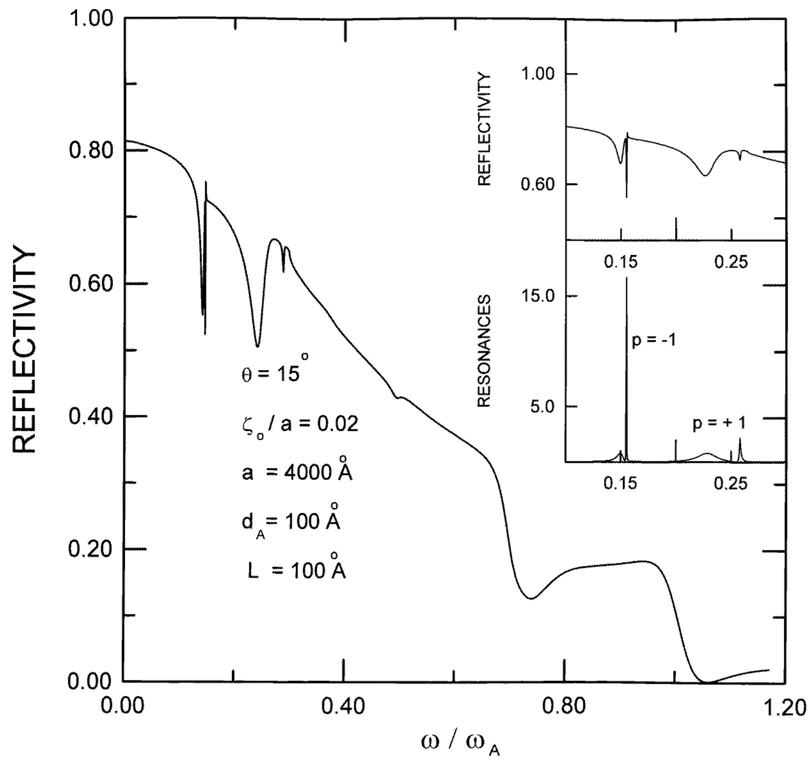

Figure 8. The reflectivity of the p-polarized waves incident on the bimetallic arrangement made up of a Mg corrugated film and a flat Al film, with the resonances $|R_{+1}|^2$ and $|R_{-1}|^2$ shown in the inset. The period of the grating is $a = 4000$ Å; $\theta = 15°$, $\xi_0/a = 0.02$, $d_A = 100$ Å, $L = 100$ Å.

[16]. Since we are dealing with a symmetric profile, we have obtained corroboration for the corresponding feature by calculating the reflectivity for small angles of incidence, using the following parameters: $\xi_0/a = 0.01$, $L_A = 100$ Å, $L_B = 200$ Å, and $L = 100$ Å; see figure 7. Above $\theta = 0.35°$, $|R_0|^2$ exhibits two minima, but below this angle, there is only one minimum, indicating that the minigap vanishes with decreasing angle of incidence. In fact, we have obtained minigaps of the order of 18 meV for near-normal incidence. As the height of the roughness is increased, the minigap size varies with a nonmonotonic behaviour. Our calculations for $\xi_0/a = 0.03$ and 0.05 yield minigaps of 20 meV and 37 meV, respectively. According to Barnes *et al* [17], it is possible to remove the degeneracy for normal incidence of light if one considers profiles with odd parity, or a linear combination of even functions centred at different positions.

In figure 8 we display the reflectivity of the bimetallic system. We see that there are four minima in the region where the surface polaritons appear, and a minimum in the interface plasmon region. An inset is included to show $|R_0|^2$ (upper panel), and the corresponding resonances $|R_{+1}|^2$ and $|R_{-1}|^2$ (lower panel) due to the surface plasmon excitations with k_{+1} and k_{-1}, respectively. The minima at $\omega = 0.155\omega_A$ and $0.2581\omega_A$ result from the coupling of the incident light with the surface mode via the sinusoidal grating, and resemble the symmetric and antisymmetric modes emerging from the plasmon coupling of the two bimetallic free surfaces [4]. These modes exist in the system because of the negative

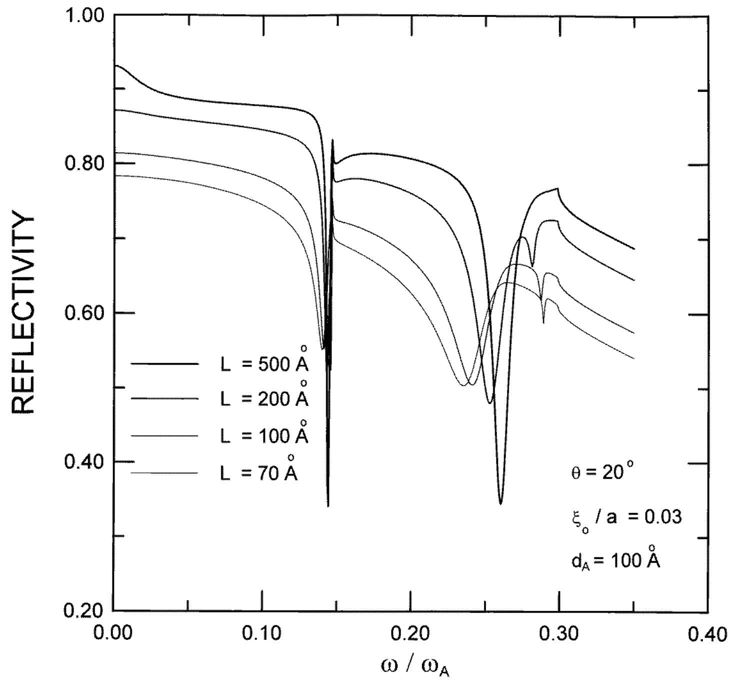

Figure 9. Low-frequency minima of the reflectivity shown in figure 8 corresponding to the 'excitation' of surface plasmons, for different thicknesses of the corrugated Mg film, $L = 70$ Å, 100 Å, 200 Å, 500 Å; $\theta = 20°$; $\xi_0/a = 0.03$; $d_A = 100$ Å.

sign of the effective dielectric function, and tend to disappear as the thicknesses of the components increase, as can be seen in figure 9. The other two minima with $\omega = 0.15\omega_A$ and $0.225\omega_A$ correspond to the excitation of the surface plasmon–polariton modes. In the range $0.7\omega_A < \omega < \omega_A$, the dielectric functions of the two metals have opposite signs; furthermore, they satisfy the condition for the existence of interface modes. The results for the reflectivity show that, in fact, these modes may be excited at the interface of the Al–Mg layers.

Finally, we explore the dependence of the surface plasmon modes on the film thicknesses and the corrugation strength. Figure 9 depicts the evolution of the reflectivity minima for different values of the thicknesses. As expected, the minima of the surface modes that resemble the symmetric and antisymmetric modes degenerate into the single surface modes as the thicknesses increase. The behaviour of $|R_0|^2$ as a function of the corrugation strength is shown in figure 10. As the amplitude of the grating ξ_0 increases, the minima corresponding to the surface modes become deeper, indicating a stronger coupling with the incident light, and consequently it becomes easier to detect them.

4. Conclusions

In this work, we have studied the scattering of light incident on a sinusoidal grating surface of a film in contact with a semi-infinite bimetallic superlattice. Investigations have been performed applying the Rayleigh–Fano approach as well as the transfer-matrix theory. Using the minima of the reflectivity $|R_0|^2$, induced by the coupling of light with the plasmons at

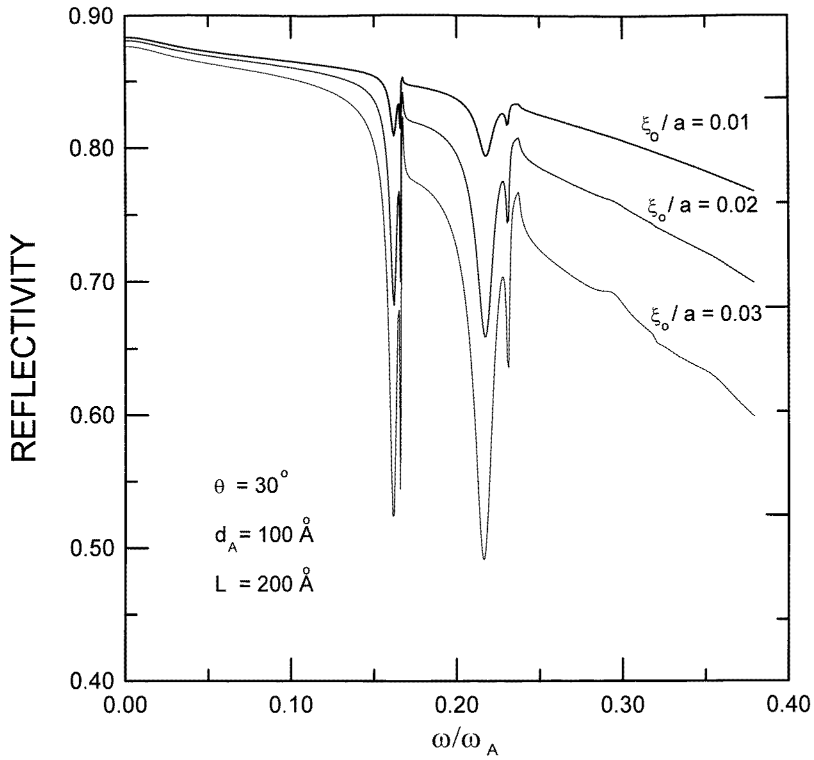

Figure 10. The dependence of the reflectivity spectrum on the corrugation strength. Here we considered $\xi_0/a = 0.01, 0.02, 0.03$.

the free surface of the metallic film, we have constructed the 'optical' dispersion relation for the surface plasmons which shows a strong dependence on the corrugation as well as on the film thickness. As the corrugation strength increases, within the Rayleigh–Fano regime, the minima of $|R_0|^2$ become deeper, indicating a stronger coupling of light with the surface plasmons. The reflectivity of the bimetallic system exhibits additional structure as produced by the coupling of the plasmons of the surfaces, resembling the 'symmetric' and 'antisymmetric' modes. The results for the collective-normal-mode dispersion relation of the infinite bimetallic superlattice show, for $p = 0$, two branches of the interface plasmon–polaritons of the successive adjacent layers in the range $\omega_B \leqslant \omega \leqslant \omega_A$, corresponding to the minima obtained in the reflectivity in the same range of energy.

Acknowledgment

This work was partially supported by CONACyT-México under contract No 481100-5-5264E.

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